Semi-group theory for the Stokes operator with Navier-type boundary conditions on $\boldsymbol{L}^p$-spaces

Chérif AMROUCHE, Université de PAU et des Pays de l'Adour

The aim of this work is to study the analyticity of the Stokes operator with Navier or Navier-type boundary conditions on $\boldsymbol{L}^p$-spaces in order to get strong, weak and very weak solutions to the following initial boundary Stokes problem:

$$\begin{cases} \frac{\partial \boldsymbol{u}}{\partial t} - \Delta \boldsymbol{u} + \nabla \pi = \boldsymbol{f}, \quad \operatorname{div} \boldsymbol{u} = 0 & \text{in} \quad \Omega \times (0,T), \\ \boldsymbol{u}(0) = \boldsymbol{u}_0 & \text{in} \quad \Omega, \end{cases} \tag{1}$$

with the following Navier or Navier-type boundary condition:

$$\boldsymbol{u} \cdot \boldsymbol{n} = 0, \qquad [\mathbf{D}(\boldsymbol{u})\boldsymbol{n}]_{\boldsymbol{\tau}} = \mathbf{0} \quad \text{on } \Gamma \times (0,T). \tag{2}$$

$$\boldsymbol{u} \cdot \boldsymbol{n} = 0, \qquad \mathbf{curl}\, \boldsymbol{u} \times \boldsymbol{n} = \mathbf{0} \quad \text{on } \Gamma \times (0,T), \tag{3}$$

In this work we prove that the Stokes operator with Navier-type boundary conditions generates a bounded analytic semi-group on the space

$$\boldsymbol{L}^p_{\sigma,T}(\Omega) = \left\{ \boldsymbol{v} \in \boldsymbol{L}^p(\Omega);\ \operatorname{div} \boldsymbol{v} = 0 \text{ in } \Omega \quad \text{and} \quad \boldsymbol{v} \cdot \boldsymbol{n} = 0 \text{ on } \Gamma \right\}.$$

The idea is to study the resolvent of the Stokes operator:

$$\lambda \boldsymbol{u} - \Delta \boldsymbol{u} + \nabla \pi = \boldsymbol{f}, \quad \operatorname{div} \boldsymbol{u} = 0 \qquad \text{in } \Omega, \tag{4}$$

with the boundary conditions (3) or (2), and where $\lambda \in \mathbb{C}^*$ satisfies $\operatorname{Re} \lambda \geq 0$. We prove the existence of weak, strong and very weak solutions to Problem (4),(3) or Problem (4),(2) satisfying the following resolvent estimate

$$\|\boldsymbol{u}\|_{\boldsymbol{L}^p(\Omega)} \leq \frac{C(\Omega, p)}{|\lambda|} \|\boldsymbol{f}\|_{\boldsymbol{L}^p(\Omega)}.$$

We study also the boundedness of the pure imaginary powers of the Stokes operator.

References

[1] Al Baba, H., Amrouche, C., Escobedo, M. On the analyticity of the semigroup generated by the Stokes operator with Navier boundary condition, *in progress.*

[2] Amrouche, C., Penel, P., Seloula, N.E.H. Some remarks on the boundary conditions in the theory of Navier-Stokes equations, *Les Annales Scientifiques Blaise Pascal*, **20-1**, 133–168, (2013).

[3] Amrouche, C., Seloula, N.E.H. Theory for vector potentials and Sobolev's inequalities for vector fields. Application to the Stokes equations with pressure boundary conditions, *Math. Mod. Meth. Appl. Sc.*, **23-1**, 37–92, (2013).

[4] Mitrea, M., Monniaux, S. On the analyticity of the semigroup generated by the Stokes operator with Neumann-type boundary conditions on Lipschitz subdomains of Riemann manifolds, *Trans. AMS*, Vol. **261-6**, (2009), 3125–3157.