ANALYSIS OF THE EXISTENCE FOR NAVIER-STOKES EQUATIONS WITH ANISOTROPIC DIFFUSION

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In this talk we consider the Navier-Stokes problem with anisotropic diffusion in a bounded domain $\Omega \subset \mathbb{R}^N$, $N \geq 2$, with a Lipschitz-continuous boundary $\partial\Omega$. The governing equations of the problem are:

$$\operatorname{div} \mathbf{u} = 0 \quad \text{in} \quad \Omega; \tag{0.1}$$

$$\mathbf{div}(\mathbf{u} \otimes \mathbf{u}) = \mathbf{f} - \nabla p + \sum_{i=1}^{N} D_i \left(|D_i \mathbf{u}|^{q_i - 2} D_i \mathbf{u} \right) \quad \text{in} \quad \Omega; \tag{0.2}$$

$$\mathbf{u} = \mathbf{0} \qquad \text{on} \quad \partial\Omega. \tag{0.3}$$

Here, $\mathbf{u} = (u_1, \cdots, u_N)$ is the velocity field, p stands for the pressure divided by the constant density and $\mathbf{f} = (f_1, \cdots, f_N)$ is the external forces field, $D_i \mathbf{u} = (\partial_i u_1, \ldots, \partial_i u_N)$ and $\partial_i u_j = \frac{\partial u_j}{\partial x_i}$. The exponents q_i are assumed to be constants with possible distinct values but such that $1 < q_i < \infty$ for any $i = 1, \ldots, N$.

For this problem we prove the existence of weak solutions in the sense that solutions and test functions are considered in the same admissible space. We prove also the existence of very weak solutions, *i.e.*, solutions for which the test functions have more regularity. By exploiting several examples we show, in the case of dimension 3, that these existence results improve its isotropic versions in almost all directions or for particular choices of all the diffusion coefficients. Principal results of this talk are published in [1].

Joint work with Hermenegildo Borges de Oliveira.

REFERENCES

1. Antontsev S., H.B.de Oliveira, *Analysis of the existence for the steady Navier- Stokes equations with anisotropic diffusion,* Advances in Differential Equations, Volume 19, Numbers 5-6(2014), 441-472.

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