

# Generalized stochastic flows and applications to incompressible viscous fluids

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## Abstract

We introduce a notion of generalized stochastic flows on manifolds, that extends to the viscous case the one defined by Brenier in [1] for perfect fluids. Their kinetic energy extends the classical kinetic energy to Brownian flows, defined as the  $L^2$  norm of their drift. We prove that there exists a generalized flow which realizes the infimum of the kinetic energy among all generalized flows with prescribed initial and final configuration. We also construct generalized flows with prescribed drift and kinetic energy smaller than the  $L^2$  norm of the drift.

The results are actually presented for general  $L^q$  norms, thus including not only the Navier-Stokes equations but also other equations such as the porous media.

## References

- [1] Brenier, Y. “The least action principle and the related concept of generalized flows for incompressible perfect fluids,” J. Amer. Math. Soc. Vol 2, no. 2 (1989), 225–255.