

Non-Isothermal Navier-Stokes System with Tresca's Friction Law: Existence, Uniqueness and Regularity Properties

Mahdi Boukrouche¹, Imane Boussetouan^{1,*}, Laetitia Paoli¹

¹ Institut Camille Jordan, Université Jean Monnet, Saint-Etienne, France
*imane.boussetouan@univ-st-etienne.fr

Abstract

Motivated by extrusion problems, we consider an unsteady fluid flow in the three-dimensional case with a temperature dependent viscosity. The problem is described by the Navier-Stokes system subjected to non-homogeneous Dirichlet conditions on a part of the boundary and *Tresca's friction law* [3] on the other part. We construct a sequence of approximate problems by using a regularization of the free boundary condition due to friction combined with a special penalty method [4] in order to get more insights on the pressure field. We prove the existence of solutions of the approximate problem by the Galerkin method. Then, we pass to the limit by using some compactness arguments to obtain a solution to our original problem [1]. We establish the uniqueness of the solution under some additional assumptions on the data, leading to better regularity properties for the velocity and pressure fields and also for the stress tensor [2].

References

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