# Lagrangian methods: Uniqueness and Analyticity

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**Abstract**

The talk is devoted to two results, one about uniqueness of solutions and the other about analyticity of Lagrangian paths for many hydrodynamic models. Uniqueness of solutions of complex hydrodynamic systems can be difficult to prove in low regularity situations. A Lagrangian description allows a robust uniqueness proof strategy in spaces that guarantee spatial Lipschitz continuity of velocities. In the same class of velocities, in joint work with Vlad Vicol and Jiahong Wu, we prove that particle paths are real analytic in time. We discuss general incompressible inviscid models, including the Euler equations, the surface quasi-geostrophic equation, incompressible porous medium equation, complex fluids, and Boussinesq equations. All these models have classical unique solutions, at least for short time. We show that they have real analytic Lagrangian paths. More precisely, we show that as long as a solution of any of these equations is in a class of regularity that assures Hölder continuous gradients of velocity, the corresponding Lagrangian paths are real analytic functions of time. The method of proof is conceptually straightforward and general, and we address the combinatorial issues head-on.