On the Existence of Singular Solutions to the Stokes Problem in the Power Cusp Domains

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Abstract

We consider initial boundary value problem for the nonsteady Stokes equations

$$\begin{cases} \frac{\partial \mathbf{u}}{\partial t} - \nu\Delta\mathbf{u} + \nabla p = \mathbf{f}, \quad x \in \Omega, \\ \operatorname{div}\mathbf{u} = 0, \quad x \in \Omega, \\ \mathbf{u}|_{\partial\Omega} = 0, \quad \mathbf{u}(x,0) = \mathbf{a}(x), \end{cases}$$

in the power cusp domains

$$\Omega = \left\{ x \in \mathbb{R}^n : \, |x'| < \varphi(x_n), \, x_n \in (0,T) \right\},$$

where $\varphi(x_n) = \mathrm{const} \cdot x_n^{\lambda}$, $\lambda > 1$, $x' = (x_1, ..., x_{n-1})$, $n = 2, 3$, and we look for the solutions satisfying the additional nonzero flux condition

$$\int\limits_{\sigma(t)} \mathbf{u} \cdot \mathbf{n} \, ds = F(t)$$

with

$$\sigma(t) = \{ x \in \Omega : \, x_n = t = \mathrm{const} \}.$$

The initial value vector $\mathbf{a}$ is assumed to have the form

$$\mathbf{a}(x', x_n) = x_n^{-\lambda(n-1)} \mathbf{a}\left(\frac{x'}{x_n}\right).$$

The complete asymptotic expansion near the singularity point is constructed and justified. The solution to the nonsteady Stokes problem is then constructed as a sum of asymptotic and other, better-behaved, terms.