On a stochastic Leray-α model of Euler equations

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Abstract

To analyze the Navier-Stokes equations, Leray [2] introduced the following regularized system

$$\begin{cases} \partial_t v - \nu \Delta v + (u \cdot \nabla) v + \nabla p = f \\ u = (1 - \alpha \Delta)^{-1} v \\ \text{div } v = 0 \end{cases} \tag{1}$$

for any $\alpha > 0$.

We are interested in the inviscid case ($\nu = 0$) for which the well posedness is not known; we add a random perturbation

$$\begin{cases} dv + [(u \cdot \nabla) v + \nabla p] \, dt = ((\sigma \circ dW) \cdot \nabla) v \\ u = (1 - \alpha \Delta)^{-1} v \\ \text{div } v = 0 \end{cases} \tag{2}$$

and for this system we proved in [1] that there exists a unique (in law) global solution. The proof uses Girsanov transform. It is crucial to add the random perturbation in the Stratonovich form in such a way that formally the energy of the vector field v is conserved. The result holds for the initial velocity of finite energy and the solution has finite energy a.s. All our results are stated for a three dimensional spatial domain (a box $[0, 2\pi]^3$, assuming periodic boundary conditions), but our proofs can be easily adapted to the two dimensional case.

References

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