On the uniqueness of stationary measure for the stochastic system of the Lorenz model describing a baroclinic atmosphere

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Abstract

We consider the system of equations for the Lorenz model for a baroclinic atmosphere

$$\frac{\partial}{\partial t}A_1 u + \nu A_2 u + A_3 u + B(u) = g, \qquad t > 0, \tag{1}$$

on the two-dimensional unit sphere S centered at the origin of the spherical polar coordinates (λ, φ), $\lambda \in [0, 2\pi)$, $\varphi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $\mu = \sin\varphi$. Here $\nu > 0$ is the kinematic viscosity, $u(t,x,\omega) = (u_1(t,x,\omega), u_2(t,x,\omega))^{\mathrm{T}}$ is an unknown vector function and $g(t,x,\omega) = (g_1(t,x,\omega), g_2(t,x,\omega))^{\mathrm{T}}$ is a given vector function, $x = (\lambda, \mu)$, $\omega \in \Omega$, (Ω, P, F) is a complete probability space,

$$A_1 = \begin{pmatrix} -\Delta & 0 \\ 0 & -\Delta + \gamma I \end{pmatrix}, \qquad A_2 = \begin{pmatrix} \Delta^2 & 0 \\ 0 & \Delta^2 \end{pmatrix},$$

$$A_3 = \begin{pmatrix} -k\Delta & 2k\Delta \\ k\Delta & -(2k + k_1 + \nu\gamma)\Delta + \rho I \end{pmatrix},$$

$$B(u) = (J(\Delta u_1 + 2\mu, u_1) + J(\Delta u_2, u_2), \\ J(\Delta u_2 - \gamma u_2, u_1) + J(\Delta u_1 + 2\mu, u_2))^{\mathrm{T}}.$$

Also, $\gamma, \rho, k, k_1 \geq 0$ are numerical parameters, I is the identity operator, $J(\psi, \theta) = \psi_\lambda \theta_\mu - \psi_\mu \theta_\lambda$ is the Jacobi operator and $\Delta\psi = ((1-\mu^2)\psi_\mu)_\mu + (1-\mu^2)^{-1}\psi_{\lambda\lambda}$ is the Laplace-Beltrami operator on the sphere S. A random vector function $g = f + \eta$ is taken as the right-hand side of (1); here $f(x) = (f_1(x), f_2(x))^{\mathrm{T}}$ and $\eta(t,x,\omega) = (\eta_1(t,x,\omega), \eta_2(t,x,\omega))^{\mathrm{T}}$ is a white noise in t. In [1] it was obtained the sufficient conditions on the right-hand side of (1) and the parameters ν, γ, ρ, k, k_1 for existence of a stationary measure of Markov semigroup which is defined by the solutions of the Cauchy problem for (1). The uniqueness of stationary measure was obtained with the same conditions on the parameters ν, γ, ρ, k, k_1 and the additional conditions on right-hand side of (1).

References

[1] Klevtsova Yu. Yu., "On the existence of a stationary measure for the stochastic system of the Lorenz model describing a baroclinic atmosphere," Sb. Math., **204**, No. 9, 1307–1331 (2013).

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