On the least action principle for the Navier-Stokes equation

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Abstract

It is known that solutions to the Navier-Stokes equation can be related to markovian diffusions by a least action principle (see [2]). One of the aspects of these models is that the finite variation part of the motion of the fluid particle is driven by a deterministic velocity field. This latter being precisely the solution to the Navier-Stokes equation. In this talk we will extend the class of the processes which can model the solutions to the Navier-Stokes equation to a wider class of generally non-markovian stochastic processes which we will call admissible. In this case the finite variation part of the motion of the fluid particle is driven by a velocity field which is also allowed to fluctuate. Thus, each sample will be driven by its own velocity field, this latter depending on the whole history of the same noise which models the thermal effects. The least action principle of [2] is extended to this general framework which provides a necessary and sufficient condition for the mean of the random velocity fields which drives the fluid particle through the noise to be solution to the Navier-Stokes equation. One of the originality of these extensions is that a new term of pressure of purely thermal (or stochastic) origin appears which is due to the fluctuations of the drift itself. Finally an entropic approach will be discussed.

References

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