Water-wave problem for a floating structure

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Abstract

The linearized equations governing the small amplitude motion of the coupled system consisting of rigid objects floating in an incompressible inviscid liquid were derived in late 1940's by Fritz John [1, 2]. Over the next 60 years, the problem of interaction of water-waves with *fixed* submerged obstacles was thoroughly investigated but the coupled freely-floating problem was largely ignored until recently.

Trapped modes in the water-wave problem are free oscillations of the floating body with the surrounding fluid. These modes exist at discrete frequencies, mathematically corresponding to eigenvalues in the discrete spectrum or embedded in the continuous spectrum. For a freely-floating structure the modes of oscillation are characteristic of the combination of the structure and the fluid. The study of trapped modes in the water-wave problem is relevant to offshore activities and structures, think *e.g.* of oil and gas drilling, piers and other floating structures subject to tides and/or wave motions. Harbour buoys and vessels in channels and fjords are also prone to this kind of oscillations.

Unlike the water-wave trapping by fixed obstacles which corresponds to solving a linear eigenvalue problem in the frequency domain, the coupling with a freely floating object turns the problem into a quadratic operator pencil due to the additional equation governing the motion of the body itself. At the same time, one is faced with a coupled system composed of a scalar equation for the velocity potential and an algebraic system for the complex amplitude vector of the rigid body displacements.

Introducing a suitable variational and operator formulation, we show how to construct a scheme that reduces the coupled quadratic eigenvalue problem to a linear self-adjoint operator and, using variational principles, derive a condition guaranteeing the existence of trapped modes. We conclude the talk by presenting several examples of freely-floating obstacles supporting trapped modes.

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References

[1] John F., "On the motion of floating bodies. I." Comm. Pure Appl. Math., **2**, 13-47 (1949).

[2] John F., "On the motion of floating bodies. II." Comm. Pure Appl. Math., **3**, 45-101 (1950).