

# Traveling finger solutions in two-fluid flows and applications

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## Abstract

Fingering instability arises from a variety of physical processes. These fingering phenomena typically occur on the interface between two immiscible fluids in a porous medium flow or on the interface of two different phases in phase transition problems. Under external conditions (such as a pressure, the gravitational force or a heat source from far field), the interface between the fluids (or phases) tends to develop into the shape of a finger, and it penetrates into the region of the other fluid (or the other phase). The fingering phenomenon is meant to be the entire evolutionary process involving splitting and merging of finger-shaped interfaces, emerging of new fingers, vanishing of existing fingers and so on. This type of problems is interesting and important for both practical and theoretical purposes. There is a wide range of applications from the secondary oil production processes where water is injected into oil reservoir, crystal growth problems. In this talk, we discuss the existence problem of finger solutions of the Hele-Shaw model. The traveling finger solutions are travelling-wave solutions whose finger-shaped interfaces are moving along a certain direction at a constant speed. The existence of finger solutions is shown through a fixed-point argument of the Hilbert transformation [1]. Other related results and applications to mathematical biology problems will also be discussed.

## References

- [1] Su J. “On the existence of finger solutions of the Hele-Shaw equation,” *Nonlinearity*, **14**, No. 1153–166 (2001).