Grupo de Física Matemática
da Universidade de Lisboa

SEMINÁRIO DE FÍSICA-MATEMÁTICA

(em colaboração com o CMAF/UL)

Dia **19 de Dezembro de 2007** (quarta-feira), às **14h30m**, na **Sala B3-01**

"Generalized fractional evolution equation"

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Abstract

The time fractional diffusion equation is obtained from the standard diffusion equation by replacing the first-order time derivative with a fractional derivative of order $\alpha \in (0,1)$, namely

$$D_t^\alpha u(x,t) = \Delta u(x,t), \ t > 0, \ x \in \mathbb{R}, \tag{1}$$

where D_t^α is the the Riemann-Liouville or Caputo derivative of order α. The main physical purpose for investigating these type of equations is to describe phenomena of anomalous diffusion appearing in transport processes and disordered systems. All the studies done on this subject treated the finite dimensional case and sufficiently smooth initial condition. The methods used to find the explicit solution consists in applying in succession the transforms of Fourier in space and Laplace in time.

The aim of this talk is to consider the above scheme in infinite dimensions. First of all, we notice that the Gross Laplacian Δ_G is the natural generalization of the usual Laplacian to represent the diffusion in infinite dimensions. Second, the Fourier transform is replaced by the so-called Laplace transform for generalized functions. Thus the same method the solution of the problem

$$D_t^\alpha U(t) = \Delta_G U(t), \ t > 0 \tag{2}$$

is given in terms of the convolution product between the fundamental solution and the initial condition which is a generalized function. The fundamental solution is related to the Mittag-Leffler function through the Laplace transform. For the Riemann-Liouville time derivative problem, we show that the corresponding fundamental solution does not correspond to a density of a measure. Hence (2) does not admit a probabilistic interpretation. On the other hand, the same problem with the Caputo derivative may be interpreted as a probability density of a stochastic process.

First we provide the mathematical background needed to solve the problem (2). Namely, we construct the appropriate test functions space $\mathcal{F}_\theta(N'), \mathcal{G}_{\theta^*}(N)$ and the associated generalized functions $\mathcal{F}'_\theta(N')$. The elements in $F_\theta(N')$ (resp. $\mathcal{G}_{\theta^*}(N)$) are entire functions on the co-nuclear space N' (resp. in N) with exponential growth of order θ (a Young function) and of minimal type (resp. maximal type). $\mathcal{F}'_\theta(N')$ is the topological dual of $\mathcal{F}_\theta(N')$. An example of an entire function is

$$N \ni \xi \mapsto E_{\alpha,\beta}(\langle \xi, \xi \rangle) \in \mathbb{C},$$

where $E_{\alpha,\beta}$ is the Mittag-Leffler function. Then we solve the problem (2) using the introduced tools and study the stability of the solution.

Local:

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