



Grupo de Física Matemática  
da Universidade de Lisboa

## SEMINÁRIO DE FÍSICA-MATEMÁTICA

Dia 17 de Junho de 2008 (terça-feira), às 14h30m, na Sala B1-01

“Bernstein processes and optimal transport”

**Christian Léonard**

(Univ. Paris 10, Nanterre, France)

### Abstract

Consider a large number  $n$  of particles living in some state space  $\mathcal{X}$  performing independent random paths during the time interval  $[0, T]$ . At time 0 you observe a profile of their configuration close to some probability distribution  $\mu \in \mathcal{P}(\mathcal{X})$  and at the later time  $T$  you observe that their profile is close to some  $\nu \in \mathcal{P}(\mathcal{X})$ .

If  $(P_t)_{0 \leq t \leq T}$  is the semigroup associated with the dynamics of each particle, by the law of large numbers one expects to find the system with a profile close to  $\mu P_t$  at a later time  $t$ . Suppose that  $\nu$  differs from  $\mu P_T$ . This unexpected event is a large deviation from the prediction of the law of large numbers and is very rare as  $n$  is large. In 1932, Erwin Schrödinger [1] addressed the problem of finding the most likely path of the whole particle system, knowing that its initial and final profiles are respectively close to  $\mu$  and  $\nu$ . He gave the answer to this large deviation problem.

It will be shown that in the double limit of a large number  $n$  of particles and of vanishing fluctuations, the most probable path of the system solves some optimal transport problem.

### References

[1] E. Schrödinger. Sur la théorie relativiste de l'électron et l'interprétation de la mécanique quantique. *Ann. Inst. H. Poincaré*, 2:269–310, 1932.

Local:

**COMPLEXO INTERDISCIPLINAR DA  
UNIVERSIDADE DE LISBOA**

Av. Prof. Gama Pinto, 2

1649-003 Lisboa, Portugal

