Grupo de Física Matemática
da Universidade de Lisboa

SEMINÁRIO DE FÍSICA-MATEMÁTICA

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"Matrix Jacobi-Mumform systems, discrete Neumann systems on Stiefel varieties, and the addition law on Prym varieties."[1]

Yuri Fedorov
(Universitat Politecnica de Catalunya, Barcelona)

Abstract

The classical integrable Neumann system is known to be a toy model for the Jacobi-Mumford (JM) systems describing translationally invariant ows on the Jacobians of hyperelliptic curves and represented by 2×2 Lax pairs.

During the last 2 decades various generalizations of the JM systems associated with families of r-gonal curves have been constructed. There have also been several important results on discretization of these systems, which are related to addition theorems for meromorphic functions on the Jacobians of such curves.

In our talk we consider analogs of the Neumann system on the Stiefel variety $V(r,n)$, the set of $n \times r$ matrices X satisfying the constraints $X^T X = \mathbf{I}_r$. These generalizations were introduced by Reiman and Semenov.

We show that the analogs admit $2r \times 2r$ Lax representations having the block form

$$\frac{d}{dt}L(\lambda) = [L(\lambda), M(\lambda)],$$

$$L(\lambda) = \begin{pmatrix} X^T(\lambda \mathbf{I}_n - A)^{-1}P & X^T(\lambda \mathbf{I}_n - A)^{-1}X \\ \mathbf{I}_r - P^T(\lambda \mathbf{I}_n - A)^{-1}P & -P^T(\lambda \mathbf{I}_n - A)^{-1}X \end{pmatrix},$$

where $A = \operatorname{diag}(a_1, \dots, a_n)$, P is the $n \times r$-momentum satisfying the constraint $X^T P + P^T X = 0$, and λ is a rational spectral parameter.

The matrix $L(\lambda)$ is a direct generalization of the 2×2 Mumford Lax matrix for the classical Neumann system and it is a particular case of rank ρ perturbations of the constant matrix A, having the form $Y + \Sigma_{i=1}^{n} \frac{\mathcal{N}_i}{\lambda - a_i}, Y, \mathcal{N}_i \in gl(\rho)$.

General cases of rank ρ perturbations corresponding to Y with a simple spectrum and the properties of the corresponding spectral curves were studied in detail by several authors. However, our Lax matrix $L(\lambda)$ has a quite special structure, and its spectral curve $\mathcal{S}$ has extra strong singularities at the in nity. So, the previous results concerning its genus, number of in nite points, etc, should be adopted to this case.

We show that the Neumann systems on $V(r,n)$ are integrable in the non-comutative sense. Calculating the order of singularity and the genus of $\mathcal{S}$, and taking into account the involution $\sigma : \mathcal{S} \to \mathcal{S}$, we then find that the generic compex invariant manifolds of the Neumann system are open subsets of *generalized* Prym varieties $\operatorname{Prym}(\mathcal{S}/\sigma =\subset \operatorname{Jac}(\mathcal{S})$ of dimension

$$d = \frac{1}{2}\left(2r(n-r) + \frac{r(r-1)}{2} - \left[\frac{r}{2}\right]\right) + \left[\frac{r}{2}\right]$$

Explicit solution for the components of X, P in terms of theta (or sigma-) functions of $\mathcal{S}$ will be presented.

Finally, we construct a family of (multi-valued) discretizations of the Neumann system on $V(r,n)$, which preserve the first integrals and geometrically are described by translations on $\operatorname{Prym}(\mathcal{S}/\sigma)$. The latter are written explicitly in terms of $2r$ points on the curve $\mathcal{S}$.

The above results allow us to constuct other natural matrix generalizations of the Jacobi-Mumford systems linearized on Prym or generalized Prym varieties.

[1] Work in collaboration with Bozidar Jovanovic (SANU, Belgrade)

Local:
INSTITUTO PARA A INVESTIGAÇÃO INTERDISCIPLINAR DA UNIVERSIDADE DE LISBOA
Av. Prof. Gama Pinto, 2
1649-003 Lisboa, Portugal