Finite time blowup of the Fujita equation with fractional Laplacian perturbed by fractional Brownian motion

We provide conditions implying finite-time blowup of positive weak solutions to the stochastic partial differential equation

$$\begin{aligned} du(t,x) &= \left[\Delta_\alpha u(t,x) + Ku(t,x) + u^{1+\beta}(t,x)\right] dt + \mu u(t,x)\, dB_t^H, \\ u(0,x) &= f(x),\ x \in \mathbb{R}^d,\ t \geq 0, \end{aligned}$$

where $\alpha \in (0,2]$, $K \in \mathbb{R}$, $\beta > 0$, $\mu \geq 0$ and $H \in [\frac{1}{2}, 1)$ are constants, Δ_α is the fractional power $-(-\Delta)^{\alpha/2}$ of the Laplacian, (B_t^H) is a fractional Brownian motion with Hurst parameter H, and $f \geq 0$ is a bounded measurable function. To achieve this we investigate the growth of exponential functionals of the form

$$\int_{r_0}^{T} \frac{\exp(\beta(Ks + \mu B_s^H))}{s^{d\beta/\alpha}}\, ds \quad \text{as} \quad T \to \infty \quad \text{with } r_0 > 0.$$

These methods differ from those applied to equations on finite domains, where such results are usually obtained by means of the eigenvalues and the eigenfunctions of the differential operator. This is joint work with J.A. López-Mimbela and E.T. Kolkovska (Centro de Investigación en Matemáticas, Guanajuato, Mexico).