String D-branes description from 2+1D Topological Field Theory

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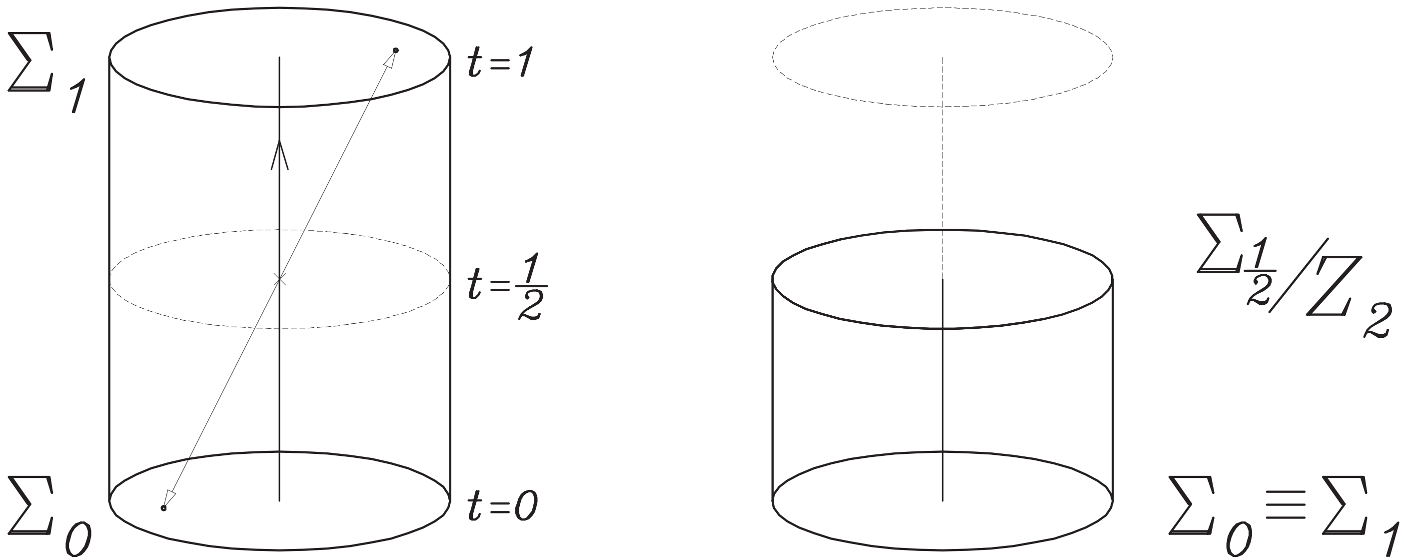
1. Topological Membrane Aproach to String Theory

- Consist in the description of String Theory by a thickened world-sheet
- This thickened world-sheet is a 2+1D Topological Membrane M with 2D time boundaries $\partial M = \Sigma_1 \oplus \Sigma_2$
- In 2+1D it is defined a Topological Field Theory and Gravity theory minimally coupled to a scalar field

  which induce in the 2D boundaries the Conformal Field Theories describing the known String Theories
- The main advantage is the reduction of fundamental principles (similarly to M-Theory)

1. Topological Membrane Aproach to String Theory

- As $\partial\partial M = 0$, open strings are described by orbifolding the membrane such that one boundary is $\Sigma_{orb} = \Sigma/\mathbb{Z}_2$



- The orbifold symmetries $\mathbb{Z}_2$ are PT (Dirichlet bc) or PCT (Neumann bc)

1. Topological Membrane Aproach to String Theory

Hence we define the 2+1 Topologically Massive Gauge Theory in $M = [0,1] \times \Sigma$

$$S_{TMGT} = \int_0^1 dt \int_\Sigma dz d\bar{z} \left[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{k}{8\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda + A_\mu J^\mu \right]$$

Canonical momenta: $\Pi^i = -F^{0i} + \frac{k}{8\pi} \epsilon^{ij} A_j$

in Schödinger picture: $\Pi^i = -i \frac{\delta}{\delta A_i}$

charge spectrum: $Q_{m,n} = m + \frac{k}{4} n$

$(z,\, \bar{z}\colon\, h_{z\bar{z}} = 1,\, \epsilon^{z\bar{z}} = i)$

1. Topological Membrane Aproach to String Theory

$$U_\Lambda = \exp\left\{ i \int_\Sigma d^2z\,\sqrt{h}\,\Lambda(\mathbf{z})\left(\partial_i E^i + \frac{k}{4\pi}B + \rho\right)\right\}$$

$$V(\mathbf{z}_0) = \exp\left\{ -i\int_\Sigma d^2z\left[\left(E^i + \frac{k\sqrt{h}}{4\pi}\epsilon^{ij}A_j\right)\epsilon_{ik}\,\partial^k \ln E(\mathbf{z},\mathbf{z}_0)\right.\right.$$
$$\left.\left. -\theta(\mathbf{z},\mathbf{z}_0)\,\rho\right]\right\}$$

$$\theta(\mathbf{z},\mathbf{z}_0) = \mathrm{Im}\,\ln\frac{E(\mathbf{z},\mathbf{z}_0)}{E(\mathbf{z},\mathbf{z}')E(\mathbf{z}',\mathbf{z}_0)}$$

$$\Big[B(\mathbf{z}), V^n(\mathbf{z}_0)\Big] = 2\pi n\,V^n(\mathbf{z}_0) \quad\Rightarrow\quad \Delta Q = -\frac{nk}{2}$$

$$\Delta Q = -\frac{nk}{2} \quad\Rightarrow\quad Q_{m,n} = m + \frac{nk}{2}$$

1. Topological Membrane Aproach to String Theory

Functional quantization follows by computing the Hamiltonean and Gauss constraints:

$$\mathcal{H}_\Sigma =$$

$$\int_\Sigma \left[\frac{1}{2}\left(\Pi^i - \frac{k}{8\pi}\epsilon^{ij}A_j\right)\left(\Pi_i - \frac{k}{8\pi}\epsilon_i{}^k A_k\right) + \frac{1}{8}(\epsilon^{ij}F_{ij})^2 - A_i J^i\right]$$

$$\mathcal{G}_\Sigma = -\partial_i\left(\Pi^i - \frac{k}{8\pi}\epsilon^{ij}A_j\right) + \frac{k}{4\pi}\epsilon^{ij}\partial_i A_j + J^0\Big|_\Sigma$$

$$\mathcal{G}_{\partial\Sigma} = \Pi^i\Big|_{\partial\Sigma}$$

$$\mathcal{H}_\Sigma\Psi[A, J] = \mathcal{E}\Psi[A, J]$$

$$\mathcal{G}_\Sigma\Psi[A, J] = \mathcal{G}_{\partial\Sigma}\Psi[A, J] = 0$$

1. Topological Membrane Aproach to String Theory

Let us consider the usual current continuity equation and a decomposition into holomorphic and anti-holomorphic components of the charge-density:

$$\partial_\mu J^\mu = 0 \quad , \quad J^0 = \rho \quad , \quad J^i = 2j^i$$

$$\rho = \frac{1}{2}\epsilon^{ij}\partial_i Y_j = i\partial_z Y^z - i\partial_{\bar{z}} Y^{\bar{z}}$$

It is further require to ensure that boundary functions are chiral, hence impose the bc:

$\Sigma_0 : \quad j^z = Y^{\bar{z}} = 0$ (anti-holomorphic)

$\Sigma_1 : \quad j^{\bar{z}} = Y^z = 0$ (holomorphic)

1. Topological Membrane Aproach to String Theory

$$\Psi_0[A,j] = \exp\left\{\int_{\Sigma_0}\left(\frac{k}{8\pi}A_{\bar z} + \frac{8\pi}{k}j^{\bar z}\right)A_z\right\}\times$$

$$\int[\mathcal{D}\varphi]\exp\left\{\frac{k}{8\pi}\int_{\Sigma_0}\left(\partial_{\bar z}\varphi - 2A_{\bar z} - \frac{64\pi^2}{k^2}j^{\bar z} + \frac{8\pi}{k}Y^z\right)\partial_z\varphi\right\}\times$$

$$\exp\left\{i\oint_{\partial\Sigma_{1.orb}}\varphi_b(Y^{\|} - A^{\|})\right\}$$

$$\Psi_1[A,j] = \exp\left\{\int_{\Sigma_1}\left(-\frac{k}{8\pi}A_z - \frac{8\pi}{k}j^z\right)A_{\bar z}\right\}\times$$

$$\int[\mathcal{D}\varphi]\exp\left\{\frac{k}{8\pi}\int_{\Sigma_1}\left(-\partial_z\varphi + 2A_z + \frac{64\pi^2}{k^2}j^z - \frac{8\pi}{k}Y^{\bar z}\right)\partial_{\bar z}\varphi\right\}\times$$

$$\exp\left\{i\oint_{\partial\Sigma_{0.orb}}\varphi_b(Y^{\|} - A^{\|})\right\},\left([D\phi] = \sqrt{\frac{\mathcal{A}_\Sigma}{det\nabla^2}}\Pi d(\partial_z\varphi)d(\partial_{\bar z}\varphi)\Pi d\varphi_b\right)$$

1. Topological Membrane Aproach to String Theory

To define the quantum field theory on an orbifold it we will consider the $\mathbb{Z}_2$ symmetries corresponding to the PT and PCT symmetries.

The specific field transformation are, for these discrete symmetries:

1. Topological Membrane Aproach to String Theory

$$PT: \quad \begin{aligned} \Lambda &\longmapsto -\Lambda \\ \varphi &\longmapsto -\varphi \\ A_0 &\longmapsto A_0 \\ A_\perp &\longmapsto A_\perp \\ A_\| &\longmapsto -A_\| \\ A_z &\longmapsto A_{\bar{z}} \\ \partial_i E^i &\longmapsto \partial_i E^i \\ B &\longmapsto B \\ Q_{m,n} &\longmapsto -Q_{m,n} \,. \end{aligned} \qquad PCT: \quad \begin{aligned} \Lambda &\longmapsto \Lambda \\ \varphi &\longmapsto \varphi \\ A_0 &\longmapsto -A_0 \\ A_\perp &\longmapsto -A_\perp \\ A_\| &\longmapsto A_\| \\ A_z &\longmapsto -A_{\bar{z}} \\ \partial_i E^i &\longmapsto -\partial_i E^i \\ B &\longmapsto -B \\ Q_{m,n} &\longmapsto Q_{m,n} \,. \end{aligned}$$

1. Topological Membrane Aproach to String Theory

To interpolate between both boundaries it is further required to describe the field and current evolution along the bulk.

Consider 2 time functions $f_0(t)$ and $f_1(t)$,

transform under PT and PCT:

$$f_0(1-t) = -f_1(t) \ ,$$

with boundary conditions

$$f_0(0) = -f_1(1) = -1 \ \ , \ \ f_0(1) = f_1(0) = 0 \ ,$$

at orbifold point

$$f_0(1/2) = f_1(1/2) = 1/2 \ .$$

1. Topological Membrane Aproach to String Theory

To extend quantities integrated at the boundaries to the bulk we consider the following construction:

$$\int_{\Sigma_1} X_1 + \int_{\Sigma_0} X_0 = \int_0^1 dt \int_\Sigma \partial_t (f_1 X_1 - f_0 X_0)$$

Such that at any time in the range $\tau \in [0,1]$ we obtain

$$\int_{\Sigma_\tau} X_\tau = \int_\Sigma (f_1(\tau) X_1 - f_0(\tau) X_0)$$

from the continuity equation

$$\partial_t (f_0 \rho_0 + f_1 \rho_1) = f_0 \partial_z j^{\bar{z}} - f_1 \partial_{\bar{z}} j^z$$

1. Topological Membrane Aproach to String Theory

we obtain the solutions

$$f_0(t) = \begin{cases} f(t) & , \quad t \in [0, 1/2[\\ -1/2 + f(t - 1/2) & , \quad t \in [1/2, 1] \end{cases}$$

$$f_1(t) = \begin{cases} 1/2 - f(3/2 - t) & , \quad t \in [0, 1/2[\\ -f(2 - t) & , \quad t \in [1/2, 1] \end{cases}$$

$$f(t) = \frac{1/2}{1 - e^{-\frac{k}{16\pi}}} + \left(1 + \frac{1/2}{1 - e^{-\frac{k}{16\pi}}}\right) e^{-\frac{k}{16\pi}\, t}$$

corresponding to the relation:

$$j^z = \frac{k}{8\pi} Y^{\bar{z}} \quad , \quad j^{\bar{z}} = \frac{k}{8\pi} Y^z$$

1. Topological Membrane Aproach to String Theory

To actually define a non chiral boundary CFT, the membrane partition function is defined as:

$$Z = \langle \Psi_1, \Psi_0 \rangle = \int [DA_z\, DA_{\bar{z}}] e^{iS_{TMGT}} \Psi_1^\dagger \Psi_0$$

While for an orbifolded membrane the partition function is:

$$Z_{orb} = \left\langle \Psi_{1/2}, \Psi_0 \right\rangle_{orb} = \int [DA_z\, DA_{\bar{z}}] e^{iS_{TMGT.orb}} \Psi_{1/2}^{orb\,\dagger} \Psi_0^{orb}$$

$$\Psi_{1/2}^{orb}[A, Y] =$$
$$\int [D\varphi] \exp\left\{ \int_{\Sigma_{orb}} \left[\left(\partial_{\bar{z}}\varphi - \frac{8\pi}{k} Y^z \right) A_z - \left(\partial_z \varphi - \frac{8\pi}{k} Y^{\bar{z}} \right) A_{\bar{z}} \right] \right\}$$
$$\times \exp\left\{ \oint_{\partial\Sigma_{orb}} \left(Y^{\parallel} - \frac{k}{8\pi} A^{\parallel} \right) \right\}$$

1. Topological Membrane Aproach to String Theory

Noting that any local gauge transformation can be included in the field φ, let us gauge fix the field A and consider the Hodge decompositions:

$$\bar{A}^i = a^i + \epsilon^{ij}\partial_j \xi \quad , \quad Y^i = \frac{k}{4\pi}\left(\partial^i Y_D + \epsilon^{ij}\partial_j Y_N\right)$$

such that these fields transform under the discrete symmetries as:

$$PT: \quad a_z \longmapsto a_{\bar{z}} \qquad\qquad PCT: \quad a_z \longmapsto -a_{\bar{z}}$$

$$Y_{\mathrm{D}} \longmapsto Y_{\mathrm{D}} \qquad\qquad Y_{\mathrm{D}} \longmapsto -Y_{\mathrm{D}}$$

$$\xi \longmapsto -\xi \qquad\qquad \xi \longmapsto \xi$$

$$Y_{\mathrm{N}} \longmapsto -Y_{\mathrm{N}} \qquad\qquad Y_{\mathrm{N}} \longmapsto Y_{\mathrm{N}}$$

1. Topological Membrane Aproach to String Theory

$$\Psi^{\dagger}_{1/2} = \int [D\varphi] \exp\left\{ -\frac{k}{8\pi} \int_{\Sigma_{orb}} \xi \nabla^2 \left(\varphi - 2Y_D\right) \right\} \Psi^{\dagger}_{\partial\Sigma_{orb}}$$

where for PT and PCT orbifolds

$$PT: \;\; \Psi^{\dagger}_{\partial\Sigma_{orb},D} = \int [D\varphi] \exp\left\{ -\frac{i\,k}{4\pi} \oint_{\partial\Sigma_{orb}} a^{\parallel} (\varphi - Y_D) \right\} \times \mathcal{V}_D$$

$$PCT: \;\; \Psi^{\dagger}_{\partial\Sigma_{orb},N} = \int [D\varphi] \exp\left\{ +\frac{i\,k}{4\pi} \oint_{\partial\Sigma_{orb}} \xi\, \partial^{\perp} \varphi \right\} \times \mathcal{V}_N$$

obtaining the EOM and BC from partition function

$$PT \quad \text{(Dirichlet bc)}: \quad \delta_{\Sigma}(\nabla^2(\varphi - 2Y_D)) \times \delta_{\partial\Sigma}(\varphi_b - Y_D)$$

$$PCT \;\; \text{(Neumann bc)}: \quad \delta_{\Sigma}(\nabla^2(\varphi - 2Y_D)) \times \delta_{\partial\Sigma}(\partial^{\perp}\varphi_b)$$

1. Topological Membrane Aproach to String Theory

in addition we obtain the D-brane and Wilson line string vertexes for the collective coordinates Y_D and Y_N:

$$PT: \ \mathcal{V}_D = \exp\left\{ -\frac{k}{4\pi} \oint_{\partial\Sigma} Y_D \partial^{\perp} \varphi_b \right\}$$

$$PCT: \ \mathcal{V}_N = \exp\left\{ -\frac{i\,k}{4\pi} \oint_{\partial\Sigma} Y_N \partial^{\parallel} \varphi_b \right\}$$

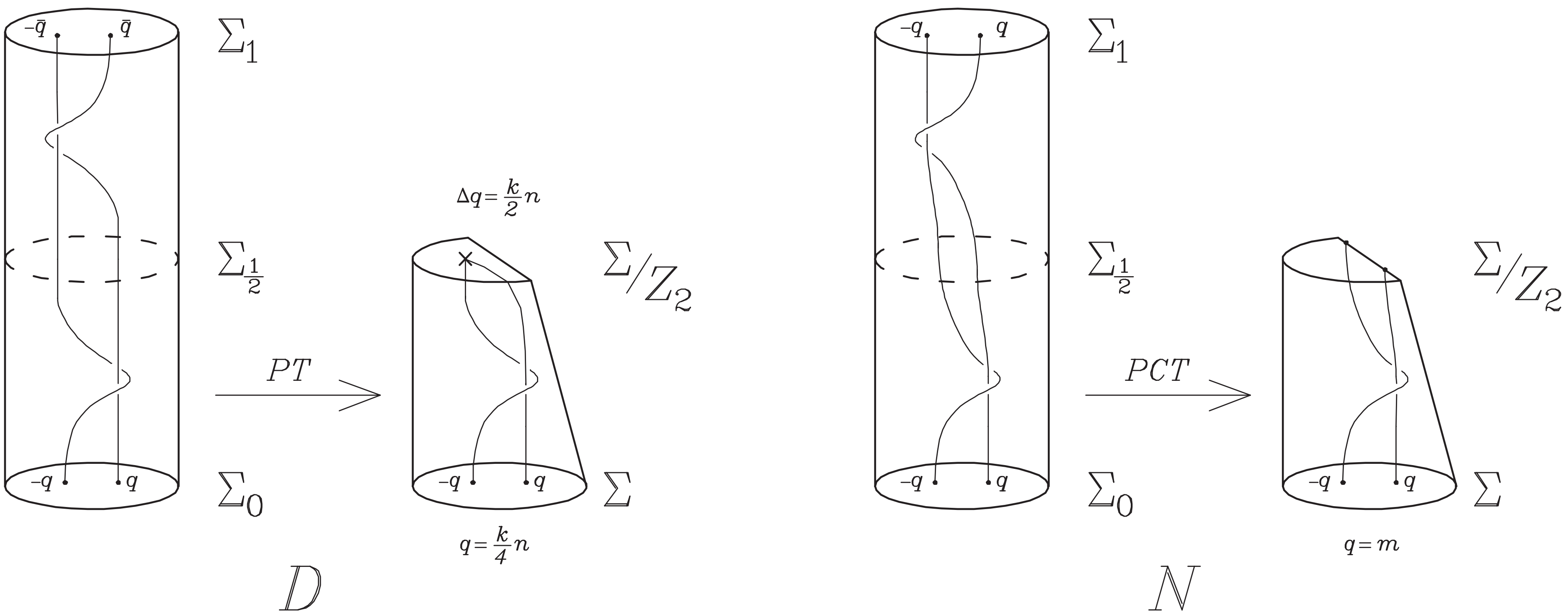
consistently also the charge spectrum is truncated by the orbifold:

$PT: \ Q_{0,n} = \dfrac{k}{4\pi} n$ (winding modes)

$PCT: \ Q_{m,0} = m$ (Kaluza-Klein modes)

identifying $k = 2R^2/\alpha'$ these correspond to 1-dimensional compact target space of string theory

1. Topological Membrane Aproach to String Theory



2. D-Branes from Topological Membrane

Aiming at describing D-branes it is required to define the respective Ishibashi states satisfying:

- Cardy condition
- sewing relations

Let us consider rational $k = \frac{2p}{q}$ (p even, q coprime)

upon identification of Σ_0 with Σ_1 the membrane describes a rational CFT.

The boundary theory will be set by the bulk topological interactions, in particular by the braiding of Wilson lines and vortex interactions (maintaining charge conservation).

2. D-Branes from Topological Membrane

The charge spectrum on Σ of genus g is given by:

$$Q^l_\lambda = \frac{\lambda^l}{q} = m^l + \frac{k}{4}n^l \quad , \quad m^l, n^l = 0, \ldots, \frac{pq}{2} - 1$$

$$\bar{Q}^l_\lambda = \frac{\lambda^l}{q} = m^l - \frac{k}{4}n^l \quad , \quad l = 1, \ldots, g$$

Let us define the punctured wave-functionals (tachions):

$$\Xi[A, Q_i(z_i)] = \Psi \times \prod_{i=1}^{s} e^{i\,Q_i(\varphi(z_i)+h_i(z_i))}$$

such that the partition function for Wilson lines (i.e. charge propagation in the bulk) is:

$$\left\langle \prod_{i=1}^{s} W_{Q_i(z,\bar{z})} \right\rangle = \Phi[A, Q_i(z_i), \bar{Q}_i(\bar{z}_i)]$$

2. D-Branes from Topological Membrane

$$\left\langle \prod_{i=1}^{s} W_{Q_i(z,\bar{z})} \right\rangle = \Phi[A, Q_i(z_i), \bar{Q}_i(\bar{z}_i)] =$$

$$\prod_{i,j=1}^{s} e^{i/k\, Q_i Q_j (\theta_{ij}(1) - \theta_{ij}(0))} \mathcal{W}_M(\Delta Q_\lambda) \Xi[A, Q_i(z_i)] \otimes \Xi^\dagger[A, \bar{Q}_i(\bar{z}_i)]$$

$$W_{Q_i(z,\bar{z})} = \exp\left\{ iQ_i \int_{C(z\rightarrow\bar{z})} A \right\}$$

$$\mathcal{W}_M(\Delta Q) = \prod_{l=1}^{g} \exp\left\{ i\Delta Q^l_\lambda \int_{\beta^l} A \right\}$$

$$\theta_{ij}(t) = \theta_i(x_i(t), x_j(t)) + 2(\Gamma^{-1})^{ll'} \mathrm{Im}\left[\int_{x'}^{x_i(0)} \omega^l \int_{x_j(0)}^{x_j(t)} (\omega^{l'} + \bar{\omega}^{l'}) + \int_{x'}^{x_j(0)} \omega^l \int_{x_i(0)}^{x_i(t)} (\omega^{l'} + \bar{\omega}^{l'}) \right]$$

2. D-Branes from Topological Membrane

Further considering sums over all possible combinations of bulk processes, considering an Hodge decomposition for the gauge fields and integrating over these fields we obtain

$$Z(\Gamma, \bar{\Gamma}) = k^{g/2} \sum_{\lambda=0}^{(pq)^g - 1} \Psi_\lambda(\Gamma) \otimes \Psi^\dagger_{\bar{\lambda}}(\bar{\Gamma})$$

Hence, by defining appropriate projection operators we can define the respective Ishibashi states:

2. D-Branes from Topological Membrane

Hence, by defining appropriate projection operators we can define the respective Ishibashi states:

$$PT: \ (Q = \bar{Q} = m) \ \ P^N_\lambda = \sum_{l=0}^{\infty} |\lambda, l\rangle \left\langle \bar{\lambda} = \lambda, l \right|$$

$$|\lambda = m\rangle^N = \sum_{l=0}^{\infty} |\lambda, l\rangle \otimes U_P \, |\lambda, l\rangle$$

$$PCT: \ (Q = -\bar{Q} = k\, n/4) \ \ P^D_\lambda = \sum_l |\lambda, l\rangle \left\langle \bar{\lambda} = -\lambda, l \right|$$

$$|\lambda = kn/4\rangle^D = \sum_{l=0}^{\infty} |\lambda, l\rangle \otimes U_P \, |\lambda, l\rangle$$

2. D-Branes from Topological Membrane

As for the description of the D-brane tension and effective actions (BI) it is required to include in the bulk:

- dilaton field D
- topologic massive gravity

$$S[A,\omega,D] = \int_M \Big[\sqrt{-g}\kappa D^2 R(\omega) + 8\sqrt{-g}\kappa\partial_\mu D\partial^\mu D$$
$$-\frac{\sqrt{-g}}{4D^2}F_{\mu\nu}F^{\mu\nu} + \frac{k}{8\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$
$$+\frac{k'}{8\pi}\epsilon^{\mu\nu\lambda}\left(\omega_\mu^a\partial_\nu\omega_\lambda^a + \frac{2}{3}\epsilon^{abc}\omega_\mu^a\omega_\nu^b\omega_\lambda^c\right)\Big] - 8\kappa\oint_{\partial M} D\partial_\perp D$$

2. D-Branes from Topological Membrane

We obtain the boundary action

$$S_b[\varphi, \phi, D] = \int_\Sigma \Big[-\frac{1}{4\pi} \left(\ln D^4 + \phi\right) R_{(2d)} - 2\kappa D \partial_\perp D + \frac{1}{16\pi} \partial_z \phi \partial_{\bar z} \phi + \Lambda_\Sigma \, e^{-\phi} - \frac{k}{8\pi} \partial_z \varphi \partial_{\bar z} \varphi \Big]$$

Hence the string tension is generated by a vev for D^4:

$$\left\langle \exp\left\{ -\frac{1}{4\pi} \int_\Sigma \left(\ln D^4\right) R_{(2d)} \right\} \right\rangle = \left\langle D^4 \right\rangle^{-\chi(\Sigma)}$$

$$\Rightarrow \quad g_s = \langle D^4 \rangle$$

$\chi(\Sigma) = \frac{1}{4\pi} \int_\Sigma R_{(2d)} = 2 - 2g - b - c$ (Euler number)

2. D-Branes from Topological Membrane

The generalization to $d + D$ dimensions is straight forward,

the bulk action is:

$$S_{TMGT}[A, D] =$$

$$\int_M \left[-\frac{\sqrt{-g}}{4D^2} F^I_{\mu\nu} F_I^{\mu\nu} - \frac{G_{IJ} + iB_{IJ}}{8\pi\alpha'} \epsilon^{\mu\nu\lambda} A^I_\mu \partial_\nu A^J_\lambda \right]$$

2. D-Branes from Topological Membrane

Further considering Neumann bc over d-dimensions and Dirichlet over D-dimensions:

$$Z^o =$$

$$(g_s)^{-\chi(\Sigma)} \prod_{I=1}^{d+D} \int [D\varphi] \prod_{a=1}^{d} \delta_{\partial\Sigma^o} \left(\varphi_b^a\right) \prod_{m=d+1}^{d+D} \delta_{\partial\Sigma^o} \left(\partial_\perp \varphi_b^m \, e^{-S_b[\varphi]}\right)$$

$$S_b[\varphi] = \frac{1}{8\pi\alpha'} \int_\Sigma \left[\sqrt{h} h^{ij} \tilde{G}_{IJ} + i\epsilon^{ij} \tilde{B}_{IJ}\right] \partial_i \varphi^I \partial_j \varphi^J +$$

$$\frac{1}{4\pi} \oint_{x^\perp=0} \left[Y_{D,a} \partial^\perp \varphi_b^a + i Y_{N,m} \partial^{\|} \varphi_b^m\right]$$

$$\tilde{K}_{IJ} = \frac{1}{(\alpha')^2} G_{II'} (K^{-1})^{I'J'} G_{J'J} \equiv \frac{1}{\alpha'} \left(\tilde{G}_{IJ} + i\tilde{B}_{IJ}\right)$$

2. D-Branes from Topological Membrane

$$S_{DBI} = Z^o = (g_s)^{-\chi(\Sigma)/2} \int d^D\varphi \left|\det\left(\mathcal{K}_{mn} + \mathcal{F}_{mn}\right)\right|^{\nu_g/2}$$

$$\mathcal{K}_{mn}(\varphi^m) = K_{IJ}\frac{\partial\varphi^I}{\partial\varphi^m}\frac{\partial\varphi^J}{\partial\varphi^n}$$

$$\mathcal{F}_{mn}(\varphi^m) = \frac{\partial Y^I_{N,m}}{\partial\varphi^n} - \frac{\partial Y^I_{N,n}}{\partial\varphi^m}$$

$$\varphi^I = (\varphi^m, Y^a_D)$$

$$\nu_0 = 0,\ \nu_1 = 1$$

$$\nu_{g\geq 2,even} = 1,\ \nu_{g>2,odd} = 0$$

Conclusions

We have described both:

- D-brane vertex operators
- D-brane effective actions