Describing Local Matter in an Expanding Universe

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1. Expansion Discovery

In 1929 was detected an isotropic systematic red-shift for radiation emitted from extragalactic sources.

It was concluded that the most probable cause would be that the universe is expanding.

At that time it was drawn the phenomenological Hubble law for distances:

$$\frac{dl}{dt} = H\, l \qquad\qquad H = 2.28 \pm 0.04 \times 10^{-18}\, s^{-1}$$

2. General Relativity Description

Soon it was derived the description of an isotropic globally flat universe within the framework of General Relativity.

For expanding coordinates the Robertson-Walker metric is:

$$ds^2 = \boxed{\left(1 - H^2 \frac{r^2}{c^2}\right)} c^2\, dt^2 + \boxed{2H \frac{r}{c}} c\, dt\, dr - dr^2 - r^2\, d\Omega$$

where the time dependent Hubble rate is related to the universe scale factor as:

$$H = \frac{\dot{a}}{a} \qquad\qquad r = l(t) = a(t)\, l_0$$

3. Problems?

Nevertheless this metric does not describe local physics.

Instead we use the Schwarzschild metric:

$$ds^2 = Z\ c^2\, dt^2 - \frac{1}{Z}\ dr^2 - r^2\, d\Omega$$

$$Z = 1 - U = \left(1 - \frac{2GM}{c^2\, r}\right)$$

How to describe both global expantion and local physics?

4. A Technical Solution

Noting that: $\lim_{r \to +\infty} Z = 1$

Define the shift function: $N_r = H \frac{r}{c} \times Z^{\frac{\alpha}{2}+\frac{1}{2}}$

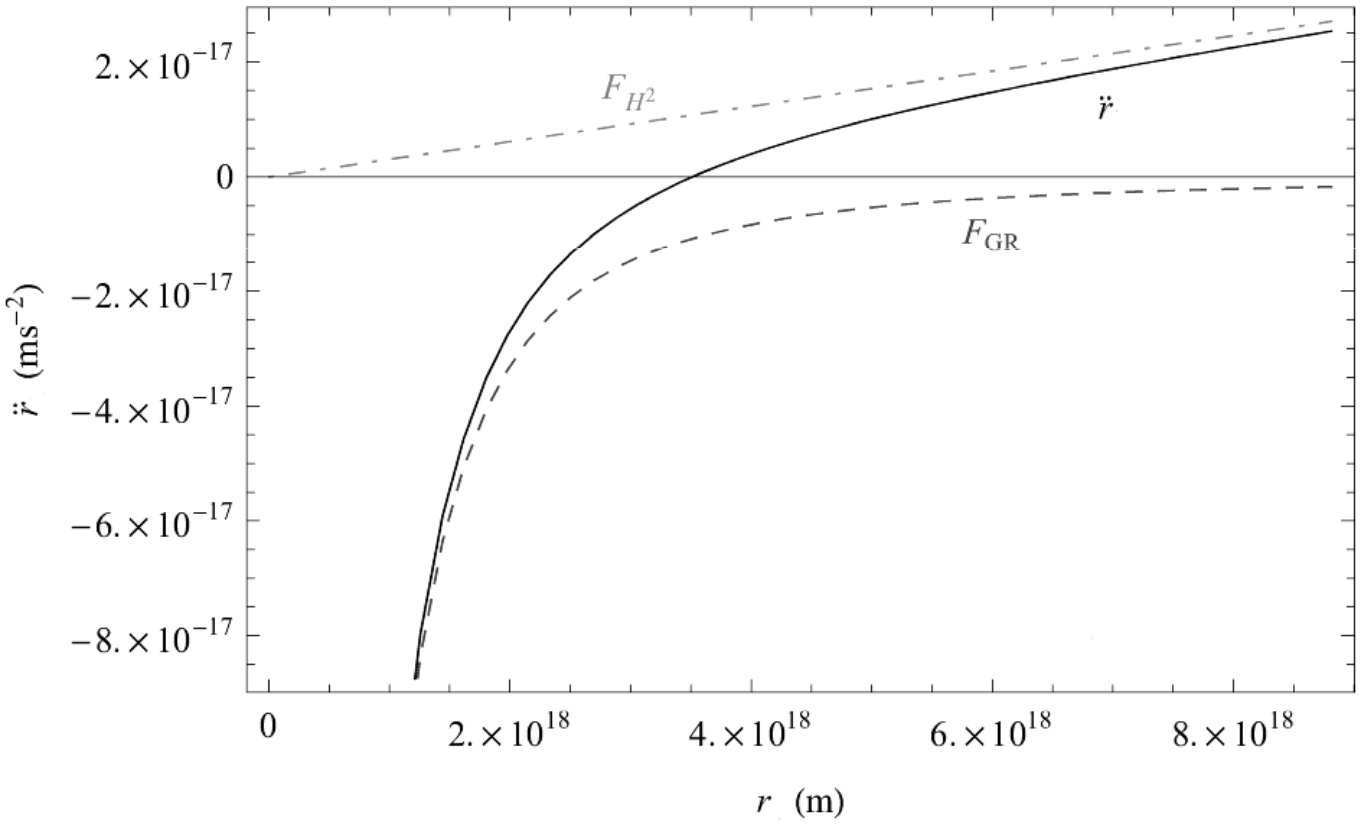
such that RW metric is recovered at spatial infinity.

Further demanding the measure of both RW and SC metric to be mantained: $\sqrt{-g} = r^2 \sin\theta$, we obtain ($\alpha \geq 3$):

$$ds^2 = \left(Z - H^2 \frac{r^2}{c^2} Z^{\alpha} \right) c^2 dt^2 + 2H \frac{r}{c} Z^{\frac{\alpha}{2}-\frac{1}{2}} c\, dt\, dr - \frac{1}{Z} dr^2 - r^2 d\Omega$$

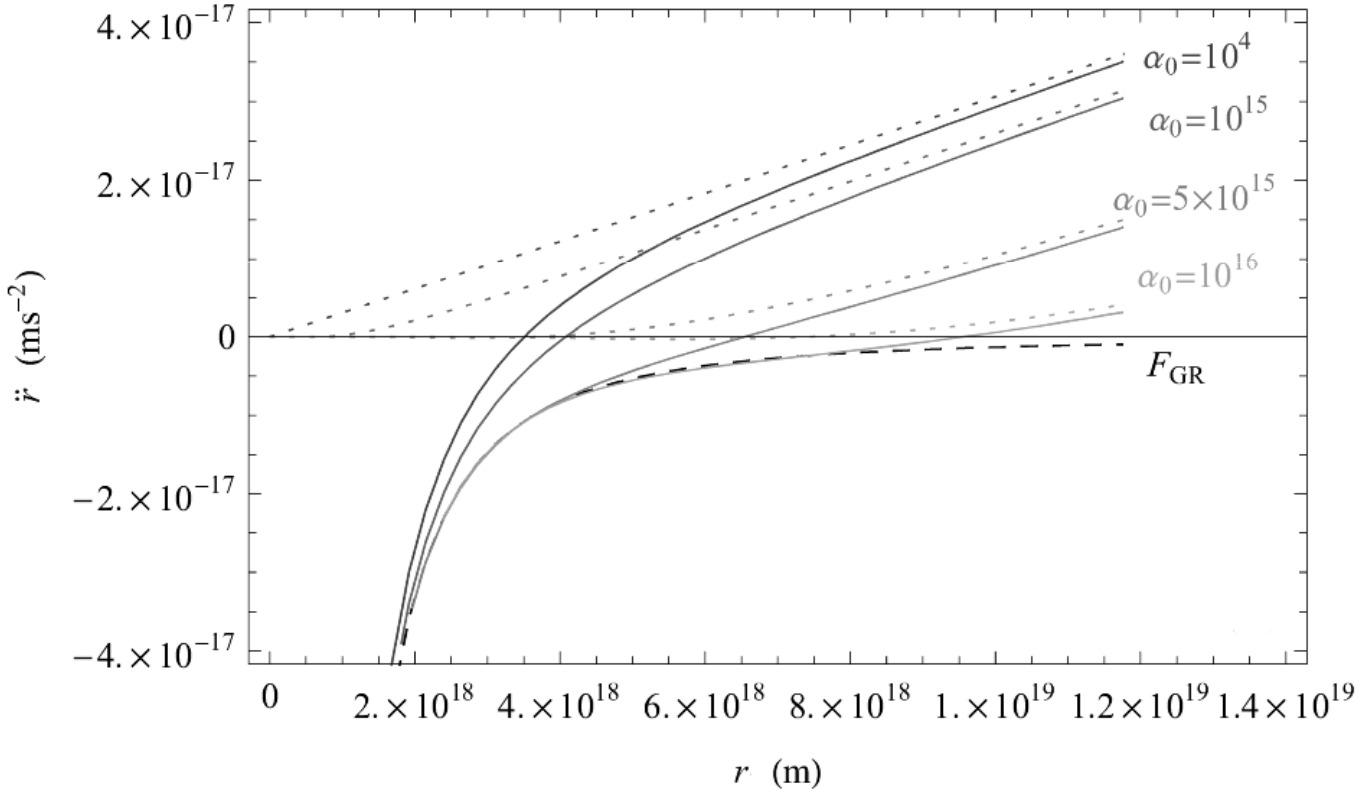
5. The Physical Consequences?

Newton law for the Sun:



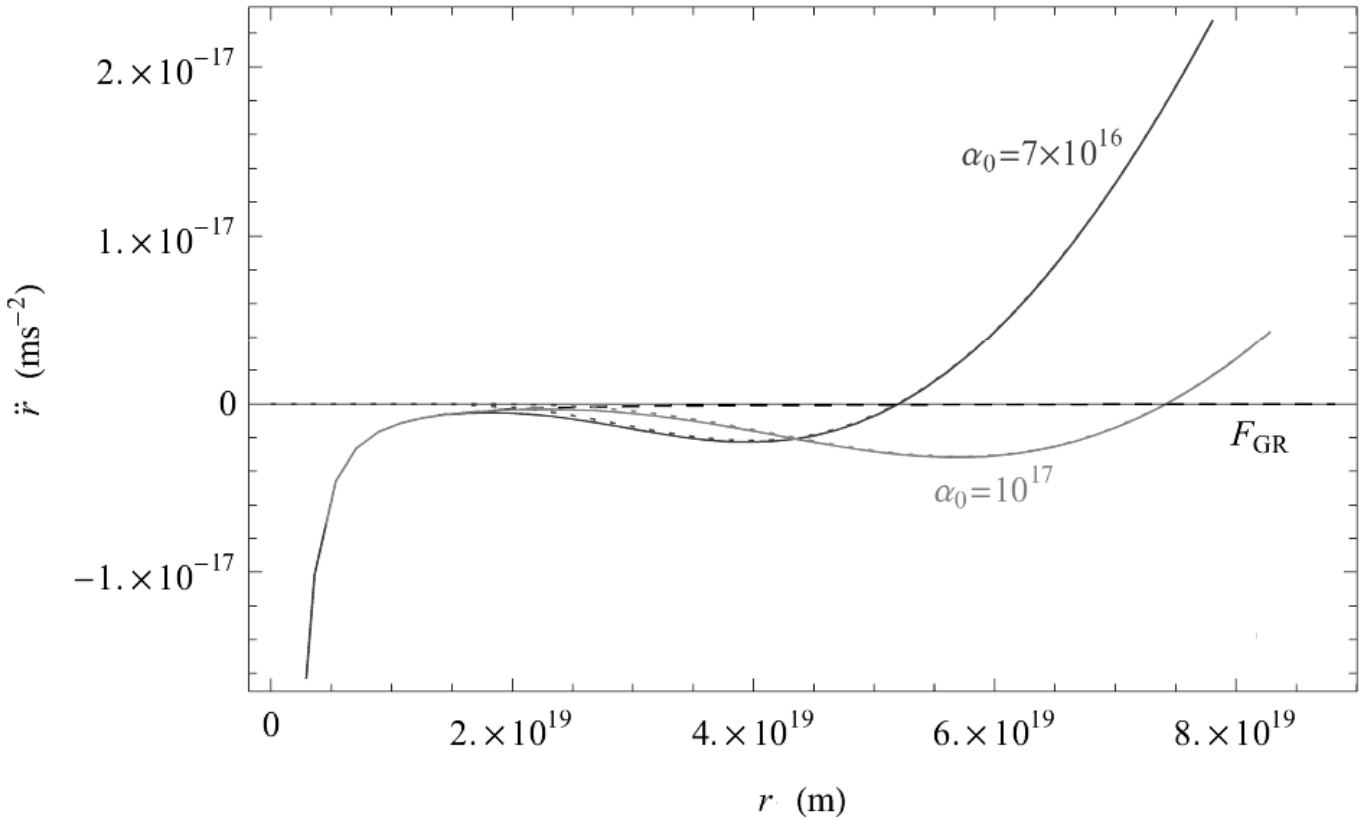
5. The Physical Consequences?

Invertion distance changes with α:



5. The Physical Consequences?

For high enough α there is an inflection:



is expansion contributing to Dark Matter effects in galaxies?